8-3 Slope of a Line

Objective: To find the slope of a line.

Vocabulary

Slope If (x_1, y_1) and (x_2, y_2) are *any* two different points on a line,

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{difference between } y\text{-coordinates}}{\text{difference between } x\text{-coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Positive slope The slope of a line that rises from left to right is positive.

Negative slope The slope of a line that falls from left to right is negative.
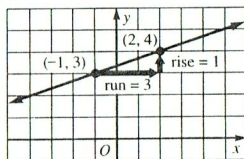
Zero slope A horizontal line has slope 0.

No slope A vertical line has no slope.

Collinear points Points that lie on the same line.

Example 1   Find the slope of the line through $(-1, 3)$ and $(2, 4)$.

Solution   Let $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (2, 4)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{2 - (-1)} = \frac{1}{3}$$

Example 2   Find the slope of the line through $(1, -3)$ and $(4, -3)$.

Solution $\text{Slope} = \frac{-3 - (-3)}{4 - 1} = \frac{0}{3} = 0$ The line has slope 0.

Example 3   Find the slope of the line through $(2, -1)$ and $(2, 5)$.

Solution $\text{Slope} = \frac{5 - (-1)}{2 - 2} = \frac{6}{0}$ (undefined)   The line has *no* slope.

Find the slope of the line through the given points.

1. $(5, -6), (2, -4)$ $-\frac{2}{3}$
2. $(-3, 6), (-5, 4)$ **1**
3. $(0, 1), (2, -2)$ $-\frac{3}{2}$
4. $(1, 2), (4, 6)$ $\frac{4}{3}$
5. $(2, 1), (8, -2)$ $-\frac{1}{2}$
6. $(-1, 5), (0, 0)$ -5
7. $(4, 3), (2, 7)$ -2
8. $(5, 2), (-1, 2)$ **0**
9. $(-3, -4), (1, 2)$ $\frac{3}{2}$
10. $(-5, 2), (7, -6)$ $-\frac{2}{3}$
11. $(1, 4), (-3, 0)$ **1**
12. $(4, 4), (-4, 6)$ $-\frac{1}{4}$
13. $(8, -1), (6, 0)$ $-\frac{1}{2}$
14. $(3, -1), (-2, 4)$ -1
15. $(7, 4), (7, -4)$ **no slope**

8-3 Slope of a Line (continued)

Example 4   Find the slope of the line with the equation $2x + 3y = 6$.

Solution   1. First find any two points on the line.

If $x = 0$:	$2(0) + 3y = 6$	If $y = 0$:	$2x + 3(0) = 6$
	$3y = 6$		$2x = 6$
	$y = 2$		$x = 3$
One point:	$(0, 2)$	Another point:	$(3, 0)$

2. Now use the slope formula. $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{3 - 0} = -\frac{2}{3}$

Find the slope of each line. If the line has no slope, say so.

16. $y = 2x - 1$ **2**
17. $y = 3x + 2$ **3**
18. $y = 4 - 2x$ -2
19. $y = 6 - 3x$ -3
20. $6x + 2y = 3$ -3
21. $2x - 5y = 10$ $\frac{2}{5}$
22. $3x + 6y = 12$ $-\frac{1}{2}$
23. $x - 2y = 4$ $\frac{1}{2}$
24. $y = 5$ **0**
25. $y + 2 = 0$ **0**
26. $x = 1$ **no slope**
27. $2x - 3 = 0$
    no slope

Example 5   Draw a line through the point $P(-1, 2)$ with a slope of 3.

Solution   1. Plot point P.

2. Write the slope as $\frac{3}{1}$. Rise $= 3$. Run $= 1$.

3. From P, measure 1 unit to the right and 3 units up to locate a second point, T.

4. Draw the line through P and T.
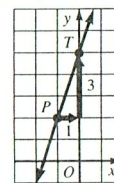
Through the given point, draw a line with the given slope.   **Graphs given at the back of this Answer Key.**

28. $A(2, 1)$; slope 2
29. $B(-2, 3)$; slope -3
30. $C(1, -4)$; slope 4
31. $D(-3, -2)$; slope $\frac{2}{3}$
32. $E(-4, 1)$: slope $-\frac{1}{2}$
33. $F(3, 0)$; slope $-\frac{3}{4}$
34. $G(-2, -1)$; slope $\frac{2}{5}$
35. $H(-5, 2)$; slope -2
36. $I(2, -3)$; slope -1

Mixed Review Exercises

Solve.

1. $\frac{x + 2}{2} + \frac{x}{4} = 0$ $\left\{-\frac{4}{3}\right\}$
2. $-3 = \frac{9b}{4}$ $\left\{-\frac{4}{3}\right\}$
3. $\frac{2 + z}{3z} = \frac{4}{z}$ $\{10\}$
4. $-3(y + 2) = 9$ $\{-5\}$

Evaluate if $x = -2$, $y = 1$, $a = 3$, and $b = -4$.

5. $\frac{a + 2b}{2a - b}$ $-\frac{1}{2}$
6. $3(x + 3y)$ **3**
7. $\frac{1}{2}(3x + 4y)$ -1
8. $(2a - 3b) + 5$ **23**

8–4 The Slope-Intercept Form of a Linear Equation

Objective: To use the slope-intercept form of a linear equation.

Vocabulary

y-intercept The y-coordinate of a point where a graph intersects the y-axis. Since the point is on the y-axis, its x-coordinate is 0.

Slope-intercept form of an equation The equation of a line in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Parallel lines Lines in the same plane that do not intersect. Lines with the same slope and different y-intercepts are parallel.

Example 1 Find the slope and y-intercept of each line: **a.** $y = \frac{5}{2}x + 4$ **b.** $y = \frac{5}{2}x$ **c.** $y = 4$

Solution Use the slope-intercept form, $y = mx + b$.

a. $y = \frac{5}{2}x + 4$

$y = \underset{\underset{m}{\uparrow}}{\frac{5}{2}}x + \underset{\underset{b}{\uparrow}}{4}$

The slope is $\frac{5}{2}$ and the y-intercept is 4.

b. $y = \frac{5}{2}x$

$y = \underset{\underset{m}{\uparrow}}{\frac{5}{2}}x + \underset{\underset{b}{\uparrow}}{0}$

The slope is $\frac{5}{2}$ and the y-intercept is 0.

c. $y = 4$

$y = \underset{\underset{m}{\uparrow}}{0}x + \underset{\underset{b}{\uparrow}}{4}$

The slope is 0 and the y-intercept is 4.

Find the slope and the y-intercept. **6.** $-\frac{1}{3}$; -3

1. $y = x - 3$ **1;** -3 **2.** $y = 2x + 3$ **2; 3** **3.** $y = -2$ **0;** -2 **4.** $y = \frac{1}{3}x + 4$ **$\frac{1}{3}$; 4**

5. $y = -\frac{1}{2}x - \frac{1}{2}$; **0** **6.** $y = -\frac{1}{3}x - 3$ **7.** $y = -2x + 6$ **-2; 6** **8.** $y = -4x + 8$ **-4; 8**

9. $y = -x + 5$ **-1; 5** **10.** $y = x - 9$ **1;** -9 **11.** $y = 3x - 2$ **3;** -2 **12.** $y = 3$ **0; 3**

Example 2 Use only the slope and y-intercept to graph $y = -\frac{2}{3}x + 4$.

Solution
1. Since the y-intercept is 4, plot $(0, 4)$.
2. Since the slope $m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{rise}}{\text{run}}$, move 3 units to the right of $(0, 4)$ and 2 units down to locate a second point.
3. Draw a line through the points.
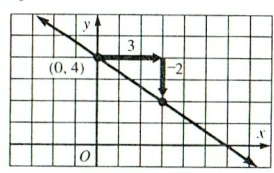
Use only the slope and y-intercept to graph each equation. You may wish to verify your graphs on a computer or a graphing calculator. **Graphs given at the back of this Answer Key.**

13. $y = \frac{2}{3}x - 4$ **14.** $y = \frac{3}{4}x - 3$ **15.** $y = -\frac{1}{2}x$ **16.** $y = -\frac{3}{4}x - 1$

17. $y = -x + 3$ **18.** $y = 2x + 1$ **19.** $y = -3$ **20.** $y = 5$

8–4 The Slope-Intercept Form of a Linear Equation *(continued)*

Example 3 Use only the slope and y-intercept to graph $2x - 3y = 6$.

Solution

$2x - 3y = 6$ Solve for y to transform the equation

$-3y = -2x + 6$ into the form $y = mx + b$.

$y = \frac{2}{3}x - 2$

1. Since $b = -2$, plot $(0, -2)$.
2. Since $m = \frac{2}{3}$, move 3 units to the right and 2 units up to locate a second point.
3. Draw a line through the points.
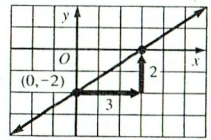
Use only the slope and y-intercept to graph each equation. You may wish to verify your graphs on a computer or a graphing calculator. **Graphs given at the back of this Answer Key.**

21. $2x + y = 4$ **22.** $3x + y = 6$ **23.** $2x - y = -6$ **24.** $3x - y = 3$

25. $x + 2y = -2$ **26.** $2x - 3y = 6$ **27.** $4x - 3y = 12$ **28.** $x + 4y = 4$

Example 4 Determine whether the lines with equations $4x + 5y = 20$ and $4x + 5y = 10$ are parallel.

Solution Write each equation in slope-intercept form:

$4x + 5y = 20$ $4x + 5y = 10$

$5y = -4x + 20$ $5y = -4x + 10$

$y = -\frac{4}{5}x + 4$ $y = -\frac{4}{5}x + 2$

slope $= -\frac{4}{5}$ y-intercept $= 4$ slope $= -\frac{4}{5}$ y-intercept $= 2$

Since both lines have the same slope and different y-intercepts, they are parallel.

Determine whether the lines whose equations are given are parallel.

29. $2x - y = 5$
$2x - y = 8$ **yes**

30. $x - 3y = 2$
$-2x + 6y = 12$ **yes**

31. $2x - y = 6$
$2y - x = 6$ **no**

32. $3x - y = 2$
$-6x + 2y = 8$ **yes**

33. $\frac{1}{2}x - \frac{1}{2}y = 4$
$2x - 2y = 3$ **yes**

34. $4x + \frac{1}{4}y = 2$
$4x + 4y = 2$ **no**

Mixed Review Exercises

Find the slope of the line through each pair of given points. **-3**

1. $(-2, 1), (-1, 2)$ **1** **2.** $(1, 2), (3, -2)$ **-2** **3.** $(-3, 4), (-1, -2)$ **4.** $(1, 5), (2, 8)$ **3**

Factor. **$(2x + 3)(x + 2)$**   **$2(x - 1)^2$**   **$(2y + 5z)(2y - 5z)$**   **$(m - 5n)(m + 2n)$**

5. $2x^2 + 7x + 6$ **6.** $2x^2 - 4x + 2$ **7.** $4y^2 - 25z^2$ **8.** $m^2 - 3mn - 10n^2$